Math 119 **Test 4**A **Name:** Key

1. Suppose a random sample of $n = 30$ observations is selected from a population that is normally distributed with a population mean equal to 2 and a standard deviation of 0.74.

(a) (3 points) What is the mean and standard deviation of the sampling distribution?

$$\boxed{\mu = 2}$$

$$SD(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{0.74}{\sqrt{30}} \approx \boxed{0.135104898}$$

(b) (3 points) Find the probability that a sample mean exceeds 2.6.

$$P(\bar{x} > 2.6)$$

$$= P\left(z > \frac{\bar{x} - \mu}{SD}\right) = P\left(z > \frac{2.6 - 2}{0.135104898}\right) \doteq P(z > 4.44)$$

$$= \text{normalcdf}(4.44, 10) \approx \boxed{4.5 \times 10^{-6} \text{ or } \frac{45 \text{ in}}{10 \text{ million}}}$$

(c) (3 points) Find the probability that a sample mean is less than 1.

$$P(\bar{x} < 1) = P\left(z < \frac{\bar{x} - \mu}{SD}\right)$$

$$= P\left(z < \frac{1-2}{0.135104898}\right) \doteq P(z < -7.40)$$

$$= \text{normalcdf}(-10, -7.4)$$

$$= \boxed{0}$$

2. ***Hypothesis Test*** LeastWorst airline's public relations department says that the airline rarely loses passenger's luggage. It further claims that on those rare occasions when luggage is lost, 90% is recovered and delivered to its owner within 24 hours. A consumer group wants to test that claim. Assume the conditions of the Central Limit Theorem are satisfied.

(a) (2 points) State the null and alternative hypotheses. Use a one-sided, 1-Prop-Z-test.

$$\boxed{\begin{array}{l} H_0: p = 0.90 \\ H_a: p < 0.90 \end{array}}$$

(b) (2 points) The consumer group surveyed a large number of air travelers and found that only 103 of 122 people who lost luggage on LeastWorst were reunited with the missing items by the next day. What is the z-score associated with the sample proportion?

$$\hat{p} = \frac{103}{122} \quad \text{and} \quad z = \frac{\hat{p} - p}{SD(\hat{p})} \quad \text{with} \quad SD(\hat{p}) = \sqrt{\frac{pq}{n}}.$$

$$SD = \sqrt{\frac{(.9)(.1)}{122}} \approx 0.027160724 \quad \text{and} \quad \boxed{z = \frac{\frac{103}{122} - 0.90}{0.027160724} \approx -2.05}$$

(c) (2 points) Calculate the p-value.

$$\text{p-value} = P\left(\hat{p} < \frac{103}{122}\right)$$

$$= P(z < -2.05) = \text{normalcdf}(-10, -2.05)$$

$$\approx \boxed{0.0202}$$

(d) (2 points) What does the p-value represent, in words? The probability of obtaining a sample proportion as low or lower than the one obtained, 103/122, assuming $p = 0.90$. The p-val is the probability that 103 passengers or fewer out of 122 are reunited with their missing luggage within 24 hours.

(e) (2 points) State the full sentence conclusion of your test.

Since p-value < 0.05, reject H_0 and conclude that the alternative hypothesis is true. LeastWorst's 24-hr recovery rate is less than 90%

3. ***Confidence Interval*** A random sample of 920 voters in one state reveals that 506 favor approval of an issue before the legislature.

(a) (2 points) Check that the appropriate conditions for the 1-Prop-Z-Interval are satisfied.

independent trials ✓

random sample ✓

$n = 920$ less than 10% of the pop. ✓

$n\hat{p} = 920 \cdot \frac{506}{920} = 506$ and $n\hat{q} = 920 \cdot \left(1 - \frac{506}{920}\right) = 414$,

both are greater than 10.

(b) (3 points) Find the standard error of the sampling distribution of sample proportions for samples of size 920. Approximate your answer to the thousandths decimal place.

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}},$$

$$= \sqrt{\frac{\frac{506}{920} \cdot \left(1 - \frac{506}{920}\right)}{920}} = \sqrt{\frac{\frac{506}{920} \cdot \frac{414}{920}}{920}} \approx \boxed{0.016}$$

(c) (3 points) Construct the 98% confidence interval for the true proportion of all voters in the state who favor approval.

$$\boxed{(51\%, 58\%)}$$

$$\hat{p} - ME < p < \hat{p} + ME$$

$$\hat{p} - z^* \cdot SE < p < \hat{p} + z^* \cdot SE$$

$$\frac{506}{920} - 2.33(0.016) < p < \frac{506}{920} + 2.33(0.016)$$

$$\boxed{0.51272 < p < 0.58728}$$

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{337}{800} \; ; \; \hat{p}_2$$

4. ***Confidence Interval*** Independent samples of $n_1 = 800$ and $n_2 = 640$ observations were selected from binomial populations 1 and 2, and $x_1 = 337$ and $x_2 = 374$ successes were observed. Assuming the appropriate assumptions are met, so that the sampling distribution for the difference between the two sample proportions is a normal distribution.

   (a) (4 points) Find the standard error of the sampling distribution of the difference between sample proportions. Approximate your answer to the thousandths decimal place.

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \qquad 0.161734848$$

$$= \sqrt{\frac{\frac{337}{800}\left(1-\frac{337}{800}\right)}{800} + \frac{\frac{374}{640}\left(1-\frac{374}{640}\right)}{640}} \approx \boxed{0.026}$$

   (b) (4 points) Construct a 95% confidence interval for the true difference between the two population proportions

$$(\hat{p}_1 - \hat{p}_2) - ME < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + ME$$

$$(\hat{p}_1 - \hat{p}_2) - z^* \cdot SE < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z^* \cdot SE$$

$$\left(\frac{337}{800} - \frac{374}{640}\right) - 1.96(0.026) < p_1 - p_2 < \left(\frac{337}{800} - \frac{374}{640}\right) + 1.96(0.026)$$

$$-0.214085 < p_1 - p_2 < -0.112165$$

$$\boxed{-21\% < p_1 - p_2 < -11\%}$$

$$\hat{p}_1 = \frac{16}{200} = 0.08 \quad \text{and} \quad \hat{p}_2 = \frac{X_2}{n_2} = \frac{8}{200} = 0.04$$

5. ***Hypothesis Test*** A random sample of 200 bolts manufactured by a machine built by the Acme Company showed 16 defective bolts. A random sample of 200 bolts manufactured by a machine built by a competitor showed 8 defective bolts. Do these data suggest a difference in the performance of the machine types? Perform a 2-Prop-Z-Test to find out.

   (a) (2 points) State the null and alternative hypotheses.

$$H_0: p_1 - p_2 = 0$$

$$H_a: p_1 - p_2 \neq 0$$

   (b) (2 points) What is the z-score associated with the difference between the sample proportions?

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SE_{pooled}} \quad \text{with } SE_{pool} = \sqrt{\frac{\hat{p}_{pool}\hat{q}_{pool}}{n_1} + \frac{\hat{p}_{pool}\hat{q}_{pool}}{n_2}} \quad \text{and} \quad \hat{p}_{pool} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{8+16}{200+200} = \boxed{0.06}$$

$$\hat{q}_{pool} = 1 - \hat{p}_{pool} = 1 - 0.06 = \boxed{0.94}$$

$$\text{Then } SE = \sqrt{\frac{(.06)(.94)}{200} + \frac{(.06)(.94)}{200}} \approx 0.023748684 \quad \text{and}$$

$$z = \frac{0.08 - 0.04}{0.023748684} \approx 1.68$$

$$\boxed{z = 1.68}$$

   (c) (2 points) Calculate the p-value.

$$p\text{-value} = 2 \cdot P(\hat{p}_1 - \hat{p}_2 > 0.08 - 0.04) = 2 \cdot P(\hat{p}_1 - \hat{p}_2 > 0.04)$$

$$= 2 \cdot P(z > 1.68) = 2 \cdot \text{normalcdf}(1.68, 10)$$

$$\approx 0.0930$$

   (d) (2 points) State the full sentence conclusion of your test.

Since p-val > 0.05 fail to reject Ho. There is insufficient sample evidence to conclude there is a difference between the performance of the 2 machine types.

6. Citizens living near the Krusty-O factory have recently complained about the smell coming from the factory. The Environmental Protection Agency periodically samples exhaust fumes coming from factory smokestacks and measures levels of carbon dioxide and other pollutants. If they find that the proportion of pollutants is above a certain amount, say p_0, they will shut down the manufacturing process and fine the factory \$150,000. $H_0: p = p_0$ the proportion of pollutants is at an

or $H_0: p \leq p_0$ acceptable level. There is no change in

(a) (2 points) In this context, what is a Type I error? the status quo.

Factory running smoothly

Type I error wrongly reject H_0, false positive.

A type I error occurs when the proportion of pollutants is at or below the acceptable level, but test conclusions indicate otherwise.

(b) (2 points) In this context, what is a Type II error?

Type II error wrongly fail to reject H_0 when H_0 is not true.

A type II error occurs if the E.P.A. does not find the proportion of pollutants greater than p_0, when in fact it really is.

(c) (2 points) Which type of error would the factory owners, the Krusty family, consider more serious?

A type I error would be more serious to the Krustys, since that results in a shut down of the factory and a fine.

(d) (2 points) Which type of error might customers consider more serious?

A type II error. Citizens living near the factory would be more upset since the actual pollution would n't be dealt with or addressed.

7. (7 points) When a truckload of apples arrives at the Krusty-O Cereal factory, a random sample of 225 apples is selected and examined for bruises, discoloration, and other defects. The whole truckload will be rejected if more than 5% of the sample is unsatisfactory. Suppose that in fact 7.5% of the apples on the truck do not meet the desired standard. What's the probability that the shipment will be accepted anyway?

Find $P(\hat{p} < 0.05)$ if in fact $p = 0.075$.

Then,

$$P(\hat{p} < 0.05) = P\left(Z < \frac{\hat{p} - p}{SD(\hat{p})}\right) \quad \text{with } SD(\hat{p}) = \sqrt{\frac{(.075)(1-0.075)}{225}}$$

$$= P\left(Z < \frac{0.05 - 0.075}{0.017559423}\right) \qquad \approx 0.017559423$$

$$\approx P(Z < -1.42)$$

$$= \text{normalcdf}(-10, -1.42)$$

$$\doteq \boxed{0.0778}$$

Math 119 **Test 4**b **Name:** Key

1. Suppose a random sample of $n = 40$ observations is selected from a population that is normally distributed with a population mean equal to 3 and a standard deviation of .85.

(a) (3 points) What is the mean and standard deviation of the sampling distribution?

$$\mu = 3$$

$$SD(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{0.85}{\sqrt{40}} = 0.134396801$$

(b) (3 points) Find the probability that a sample mean exceeds 2.6.

$$P(\bar{x} > 2.6) = P\left(z > \frac{\bar{x} - \mu}{SD}\right) \doteq P\left(z > \frac{2.6 - 3}{0.13439680}\right)$$

$$\approx P(z > -2.98) \approx \text{normalcdf}(-2.98, 10)$$

$$\approx \boxed{100}$$

(c) (3 points) Find the probability that a sample mean is less than 2.

$$P(\bar{x} < 2) = P\left(z > \frac{\bar{x} - \mu}{SD}\right) = P\left(z > \frac{2-3}{0.13439680}\right)$$

$$= P(z > -7.44) = \boxed{0}$$

2. ***Hypothesis Test*** LeastWorst airline's public relations department says that the airline rarely loses passenger's luggage. It further claims that on those rare occasions when luggage is lost, 90% is recovered and delivered to its owner within 24 hours. A consumer group wants to test that claim. Assume the conditions of the Central Limit Theorem are satisfied.

(a) (2 points) State the null and alternative hypotheses. Use a one-sided, 1-Prop-Z-test.

$H_0: p = 0.90$ (or $p \geq 0.90$)

$H_a: p < 0.90$

p = population proportion of lost luggage that is returned within 24hrs.

(b) (2 points) The consumer group surveyed a large number of air travelers and found that only 140 of 172 people who lost luggage on LeastWorst were reunited with the missing items by the next day. What is the z-score associated with the sample proportion?

$x = 140$

$n = 172$

$\hat{p} = \frac{140}{172}$

$\hat{q} = \frac{32}{172}$

$$z = \frac{\hat{p} - p}{SD(\hat{p})}, \text{ where } \boxed{SD(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.9)(0.1)}{172}} \approx 0.022874786}$$

$$\text{So, } z = \frac{\frac{140}{172} - 0.90}{0.022874786} \approx \boxed{-3.76}$$

(c) (2 points) Calculate the p-value.

$$\text{p-value} = P\left(\hat{p} < \frac{140}{172}\right) = P(z < -3.76)$$

$$= \text{normalcdf}(-10, -3.76)$$

$$\approx \boxed{8.5 \times 10^{-5}} = 0.000085 \text{ or } 85 \text{ in } 1 \text{ million}$$

(d) (2 points) What does the p-value represent, in words?

The probability of getting a sample proportion less than or equal to $\frac{140}{172}$, in samples of size $n = 172$, assuming p really is 90%. The probability that 140 or fewer passengers have their luggage returned within 24 hours, in samples of size 172.

(e) (2 points) State the full sentence conclusion of your test.

Since p-val < 0.05 reject H_0, and conclude that the true proportion of passengers reunited with their lost luggage within 24 hours is less than 90%

$X = 506$
$n = 960$

3. ***Confidence Interval*** A random sample of 960 voters in one state reveals that 506 favor approval of an issue before the legislature.

   (a) (2 points) Check that the appropriate conditions for the 1-Prop-Z-Interval are satisfied.

   Independent trials ✓
   random Sample ✓
   $n = 960$ is less than the population ✓
   $n \cdot \hat{p} = 506$ and $n \cdot \hat{q} = 454$ both ≥ 10 ✓

   (b) (3 points) Find the standard error of the sampling distribution of sample proportions for samples of size 920. Approximate your answer to the thousandths decimal place.

   $$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{\frac{506}{960}\frac{454}{960}}{960}} \approx 0.016113739$$

   $$\approx \boxed{0.016 \text{ rounded}}$$

   (c) (3 points) Construct the 90% confidence interval for the true proportion of all voters in the state who favor approval.

   $\hat{p} - ME < p < \hat{p} + ME$

   $\hat{p} - z^* \cdot SE < p < \hat{p} + z^* \cdot SE$

   $\frac{506}{960} - 1.645(0.016) < p < \frac{506}{960} + 1.645(0.016)$

   $0.5007 < p < 0.5534$

   $p \in (50.07\%, 55.34\%)$

4. ***Confidence Interval*** Independent samples of $n_1 = 800$ and $n_2 = 640$ observations were selected from binomial populations 1 and 2, and $x_1 = 337$ and $x_2 = 374$ successes were observed. Assuming the appropriate assumptions are met, so that the sampling distribution for the difference between the two sample proportions is a normal distribution.

(a) (4 points) Find the standard error of the sampling distribution of the difference between sample proportions. Approximate your answer to the thousandths decimal place.

See the solution to problem 4 on the other test key.

(b) (4 points) Construct a 95% confidence interval for the true difference between the two population proportions

5. ***Hypothesis Test*** A random sample of 200 bolts manufactured by a machine built by the Acme Company showed 14 defective bolts. A random sample of 200 bolts manufactured by a machine built by a competitor showed 6 defective bolts. Do these data suggest a difference in the performance of the machine types? Perform a 2-Prop-Z-Test to find out.

(a) (2 points) State the null and alternative hypotheses.

$$\boxed{\begin{array}{l} H_0: p_1 - p_2 = 0 \\ H_a: p_1 - p_2 \neq 0 \end{array}}$$

$\hat{p}_1 = \frac{14}{200} = 0.07$

$\hat{p}_2 = \frac{6}{200} = 0.03$

(b) (2 points) What is the z-score associated with the difference between the sample proportions?

$$\hat{p}_{pool} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{14 + 6}{200 + 200} = \frac{20}{400} = \frac{1}{20} = 0.05; \; \hat{q}_{pool} = 0.95$$

$$SE_{pool}(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_{pool}\hat{q}_{pool}}{n_1} + \frac{\hat{p}_{pool}\hat{q}_{pool}}{n_2}} = \sqrt{\frac{(.95)(.05)}{200} + \frac{(.95)(.05)}{200}}$$

$$\approx 0.021794495$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SE} = \frac{.07 - 0.03}{0.021794495} \approx \boxed{1.84}$$

(c) (2 points) Calculate the p-value.

$$\text{p-val} = 2 \cdot P\left(\hat{p}_1 - \hat{p}_2 > 0.07 - 0.03\right) = 2 \cdot P(z > 1.84)$$

$$= 2 \cdot \text{normalcdf}(1.84, 10)$$

$$\doteq \boxed{0.0658}$$

(d) (2 points) State the full sentence conclusion of your test.

Since p-val $\approx$ 0.066 the probability of obtaining a difference in sample proportions as extreme (or more, in both directions) as the one obtained (0.04; in samples of size 200) occurs $\approx$ 6.6% of the time, assuming there is no difference. Fail to reject H_0. We can be 90% confident there is a difference in machine type performance, but not 95% confident.

6. Citizens living near the Krusty-O factory have recently complained about the smell coming from the factory. The Environmental Protection Agency periodically samples exhaust fumes coming from factory smokestacks and measures levels of carbon dioxide and other pollutants. If they find that the proportion of pollutants is above a certain amount, say p_0, they will shut down the manufacturing process and fine the factory \$150,000.

(a) (2 points) In this context, what is a Type I error?

See the solution to problem 6 on the other test key.

(b) (2 points) In this context, what is a Type II error?

(c) (2 points) Which type of error would the factory owners, the Krusty family, consider more serious?

(d) (2 points) Which type of error might customers consider more serious?

7. (7 points) When a truckload of apples arrives at the Krusty-O Cereal factory, a random sample of 225 apples is selected and examined for bruises, discoloration, and other defects. The whole truckload will be rejected if more than 5% of the sample is unsatisfactory. Suppose that in fact 10.5% of the apples on the truck do not meet the desired standard. What's the probability that the shipment will be accepted anyway?

Find $P(\hat{p} < 0.05)$ if in fact $p = 0.105$.

$$SD(\hat{p}) = \sqrt{\frac{(0.105)(1-0.105)}{225}} \approx 0.020436895$$

and

$$P(\hat{p} < 0.05) = P\left(z < \frac{\hat{p} - p}{SD(\hat{p})}\right)$$

$$= P\left(z < \frac{0.05 - 0.105}{0.020436895}\right)$$

$$\approx P(z < -2.69)$$

$$= \text{normalcdf}(-10, -2.69)$$

$$\approx 0.00357$$